FLOW AND HEAT TRANSFER IN THE HYDROMAGNETIC EKMAN LAYER ON A POROUS PLATE WITH HALL EFFECTS

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Abstract—Flow in the Ekman layer of a conducting liquid past an infinite porous plate is investigated when the liquid is permeated by a transverse magnetic field and the Hall effects are taken into account. It is found that asymptotic solution exists both in the presence of suction or blowing at the plate. For fixed magnetic and suction parameters, the skin friction of the primary flow increases with increase in the Hall parameter $\omega \tau$, while that for the secondary flow first increases, reaches a maximum and then decreases with increase in $\omega \tau$. Steady distribution of temperature in the flow exists only for suction at the plate and for large Eckert numbers, heat flows from the liquid to the wall even if the wall temperature is higher than that of the ambient stream.

NOMENCLATURE

- C_p , specific heat at constant pressure;
- e, electric charge;
- E_1 , Ekman parameter;
- *Ec*, Eckert number;

 (E_x, E_y, E_z) , components of electric field **E**; (H_x, H_y, H_z) , components of magnetic field **H**;

 \mathbf{H}_0 , applied magnetic field;

 (j_x, j_y, j_z) , components of current density **j**;

M, Hartmann number;

- n_e , number of density of electrons;
- p, pressure;
- p_e , electron pressure;
- Pr, Prandtl number;
- S, suction or blowing parameter;
- T, temperature;
- T_0 , temperature at the plate;
- T_{∞} , free-stream temperature;
- (u, v, w), components of velocity field **q**;
- U_{∞} , free-stream velocity;
- (x, y, z), Cartesian co-ordinates.

Greek symbols

- $\omega \tau$, Hall parameter;
- v, kinematic viscosity;
- ρ , density;
- μ_e , magnetic permeability;
- Ω , angular velocity;
- ζ , dimensionless variable (zU_{∞}/v) .

INTRODUCTION

WHEN a vast expanse of viscous liquid bounded by an infinite flat plate is rotating about an axis normal to the plate, a layer is formed near the plate where the viscous and coriolis forces are of the same order of magnitude. This is known as Ekman layer (see Prandtl [1]) and the effect of a uniform transverse magnetic field on such a layer was investigated by Gupta [2]. Soundalgekar and Pop [3] extended the problem considered in [2] by assuming the plate to be porous and subjecting it to uniform suction. Hall effects and induced magnetic field were neglected in this analysis. However, when the strength of the magnetic field is very large, effects due to flow of Hall current should be taken into account (see Cowling [4]). Recently Gupta [5] investigated the flow of an electrically conducting liquid past an infinite porous flat plate in the presence of a uniform transverse magnetic field, the Hall effects being taken into account. He found that asymptotic solution for velocity exists both for suction and injection at the plate.

The purpose of the present investigation is to extend the analysis of Gupta [5] to a rotating frame of reference, and deduce the flow and heat-transfer characteristics of the Ekman layer over the plate which is maintained at a temperature higher than that of the ambient stream.

MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

Consider the steady flow of an electrically conducting liquid past an infinite porous plate when the liquid and the plate rotate in unison with a constant angular velocity Ω about z-axis taken normal to the plate upwards. A uniform magnetic field H_0 is imposed along z-axis and the plate is taken electrically nonconducting. Since the plate occupying the plane z = 0is of infinite extent, physical conditions depend on zonly in the steady state. We also assume that a uniform pressure gradient acts along y-axis so that there is a uniform flow with velocity U_{∞} along x-axis. The equation of continuity $\nabla \cdot \mathbf{q} = 0$ and the solenoidal relation for the magnetic field $\nabla \cdot \mathbf{H} = 0$ give $w = w_0 =$ constant, $H_z = \text{constant} = H_0$ everywhere in the flow. The constant w₀ representing the normal velocity at the plate is negative for suction and positive for blowing at the plate. The equation of conservation of electric charge $\nabla \cdot \mathbf{j} = 0$ gives $j_z = \text{constant}$. This constant is zero since $j_z = 0$ at the plate which is electrically nonconducting. Thus $j_z = 0$ everywhere in the flow. In a

rotating frame of reference, the equations of momentum along x, y, z directions are now given by

$$w_0 \frac{\mathrm{d}u}{\mathrm{d}z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + 2\Omega v + v \frac{\mathrm{d}^2 u}{\mathrm{d}z^2} + \frac{\mu_e H_0}{\rho} \cdot j_y \qquad (1)$$

$$w_0 \frac{\mathrm{d}v}{\mathrm{d}z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - 2\Omega u + v \frac{\mathrm{d}^2 v}{\mathrm{d}z^2} - \frac{\mu_e H_0}{\rho} \cdot j_x \qquad (2)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial z}.$$
 (3)

In deriving (1) through (3), it is tacitly assumed that the induced magnetic field is negligible so that $\mathbf{H} \equiv (0, 0, H_0)$. Such an assumption is however justified in flow of liquid metals.

When the strength of the magnetic field is very large, Ohm's law must be modified to include Hall currents as follows (see Cowling [4])

$$\mathbf{j} + \frac{\omega \tau}{H_0} \mathbf{j} \times \mathbf{H} = \sigma \left[\mathbf{E} + \mu_e \mathbf{q} \times \mathbf{H} + \frac{1}{e n_e} \nabla p_e \right]$$
(4)

where **E** is the electric field, ω is the cyclotron frequency and τ is the collision time of electrons. In writing (4), the ion slip and the thermo-electric effects are neglected and further it is assumed that $\omega_i \tau_i \ll 1$, where ω_i and τ_i are the cyclotron frequency and the collision time of ions respectively. Since the physical quantities depend on z only, equation (4) gives

$$j_x + \omega \tau j_y = \sigma [E_x + \mu_e H_0 v]$$
⁽⁵⁾

$$j_y - \omega \tau j_x = \sigma \left[E_y - \mu_e H_0 u \right] \tag{6}$$

where the electrical conductivity σ is assumed constant. The boundary conditions are

$$u(0) = v(0) = 0; \quad u \to U_{\infty} \text{ and } v \to 0 \text{ as } z \to \infty.$$
 (7)

In the free-stream, the magnetic field is uniform and $\nabla \times \mathbf{H} = \mathbf{j}$ shows that there is no electric current there. Thus

$$j_x \to 0$$
 and $j_y \to 0$ as $z \to \infty$. (8)

Using (7) and (8) in (5) and (6), we obtain

$$E_x = 0, \quad E_y = \mu_e H_0 U_\infty \quad \text{when} \quad z \to \infty.$$
 (9)

Since the pressure gradient acts along y-axis in the free stream, equations (1) and (2) give on using (3), (7) and (8)

$$\frac{\partial p}{\partial x} = 0, \quad \frac{1}{\rho} \frac{\partial p}{\partial y} + 2\Omega U_{\infty} = 0.$$
 (10)

Further since in the steady state $\nabla \times \mathbf{E} = 0$, we must have $dE_x/dz = 0$ and $dE_y/dz = 0$ and equation (9) then gives

$$E_x \equiv 0, \quad E_y \equiv \mu_e H_0 U_\infty \tag{11}$$

everywhere in the flow. Substitution of (10) in (1) and (2) gives

$$w_0 \frac{\mathrm{d}u}{\mathrm{d}z} = v \frac{\mathrm{d}^2 u}{\mathrm{d}z^2} + 2\Omega v + \frac{\mu_e H_0}{\rho} \cdot j_y \tag{12}$$

$$w_0 \frac{\mathrm{d}v}{\mathrm{d}z} = v \frac{\mathrm{d}^2 v}{\mathrm{d}z^2} - 2\Omega(u - U_\infty) - \frac{\mu_e H_0}{\rho} \cdot j_x. \quad (13)$$

Multiplying (13) by $i(=\sqrt{-1})$ and adding to (12), we get

$$w_0 \frac{\mathrm{d}}{\mathrm{d}z} (u+iv) = v \frac{\mathrm{d}^2}{\mathrm{d}z^2} (u+iv) - 2\Omega i [u-U_\infty + iv] - \frac{i\mu_e H_0}{\rho} (j_x + ij_y). \quad (14)$$

Similarly, equation (6) multiplied by i and added to (5) gives on using (11)

$$(1-i\omega\tau)(j_x+ij_y) = -i\sigma\mu_e H_0[u-U_\infty+iv]. \quad (15)$$

Eliminating $j_x + i j_y$ from (14) and (15) and introducing the dimensionless quantities

$$\overline{U} = \frac{u - U_{\infty} + iv}{U_{\infty}}, \quad \zeta = \frac{zU_{\infty}}{v}, \quad S = \frac{w_0}{U_{\infty}},$$

$$E_1 = \frac{\Omega v}{U_{\infty}^2}, \quad M^2 = \frac{\mu_e^2 H_0^2 \sigma v}{\rho U_{\infty}^2},$$
(16)

we get

$$\frac{\mathrm{d}^2 \overline{U}}{\mathrm{d}\zeta^2} - S \frac{\mathrm{d}\overline{U}}{\mathrm{d}\zeta} - \left(2iE_1 + \frac{M^2}{1 - i\omega\tau}\right)\overline{U} = 0.$$
(17)

Using (7) and (16), the boundary conditions for (17) are

$$\overline{U}(0) = -1, \qquad \overline{U}(\infty) = 0. \tag{18}$$

The solution of (17) satisfying (18) is

$$\overline{U}(\zeta) = -\exp\left[\frac{1}{2}(S - \alpha - i\beta)\zeta\right]$$
(19)

where

$$\alpha = \left[(S^2 + 4\alpha_1) + \{ (S^2 + 4\alpha_1)^2 + 4(4E_1 + 2\alpha_1\omega\tau)^2 \}^{\frac{1}{2}} \right]^{\frac{1}{2}}/2^{\frac{1}{2}}$$
(20)

$$\beta = \left[-(S^2 + 4\alpha_1) + \{(S^2 + 4\alpha_1)^2 + 4(4E_1 + 2\alpha_1\omega\tau)^2\}^{\frac{1}{2}} \right]^{\frac{1}{2}}/2^{\frac{1}{2}}$$
(21)

$$\alpha_1 = \frac{M^2}{1 + \omega^2 \tau^2}.$$
 (22)

It may be noticed from (20) that $S-\alpha < 0$ both for suction (S < 0) and blowing (S > 0) at the plate so that (19) satisfies $\overline{U}(\infty) = 0$. Thus we arrive at the interesting result that asymptotic solution for velocity exists both for suction and blowing at the plate.

For suction, we put $S = -S_1$ so that $S_1 > 0$. In this case the solution is obtained from (19) as

$$\frac{u}{U_{\infty}} = 1 - e^{-\frac{(S_1 + \alpha)\zeta}{2}} \cos\frac{\beta\zeta}{2}$$
(23)

$$\frac{v}{U_{\infty}} = e^{-\frac{(\Sigma_1 + \alpha)\zeta}{2}} \sin \frac{\beta\zeta}{2}$$
(24)

which shows that the velocity distribution is in the form of Ekman spiral.

For blowing at the plate, S > 0 and the solution is given by (19) as

$$\frac{u}{U_{\infty}} = 1 - e^{\frac{(S-\alpha)\zeta}{2}} \cos\frac{\beta\zeta}{2}$$
(25)

$$\frac{v}{U_{\infty}} = e^{\frac{(S-\alpha)\zeta}{2}} \sin \frac{\beta\zeta}{2}$$
(26)

which is also in the form of a spiral. The dimensionless skin-friction coefficients $[d/d\zeta(u/U_{\infty})]_{\zeta=0} = X$ and

 $[d/d\zeta(v/U_{\infty})]_{\zeta=0} = Y$ are evaluated from (23)–(26) as follows:

For suction (S < 0) with $S = -S_1$

$$X = \frac{1}{2}(S_1 + \alpha), \qquad Y = \frac{\beta}{2}.$$
 (27)

For blowing (S > 0)

$$X = \frac{1}{2}(\alpha - S), \qquad Y = \frac{\beta}{2}.$$
 (28)

In Table 1, we have computed X, Y and $1/X(=\delta)$ for the case of suction with $S_1 = 1$ and $E_1 = 0.5$ for various values of the Hall parameter $\omega \tau$ and two values of the Hartmann number M. It is clear from (23)–(26) that 1/X is a measure of the thickness δ of the Ekman layer. The solution of (31) satisfying (32) is

$$\theta(\zeta) = \hat{1} - e^{-S_1 \cdot Pr \cdot \zeta} + \frac{Pr \cdot Ec \cdot [(S_1 + \alpha)^2 + \beta^2]}{4(S_1 + \alpha)(S_1 \cdot Pr - S_1 - \alpha)} \times [e^{-S_1 \cdot Pr \cdot \zeta} - e^{-(S_1 + \alpha)\zeta}]$$
(33)

for $S_1 \cdot Pr - S_1 - \alpha \neq 0$, and

$$\theta(\zeta) = 1 - \left[1 + \frac{Ec\{(S_1 + \alpha)^2 + \beta^2\}}{4S_1} \cdot \zeta \right] \cdot e^{-(S_1 + \alpha)\zeta}$$
(34)

for $S_1 \cdot Pr - S_1 - \alpha = 0$.

The rate of heat transfer at the plate is given by

$$q_{w} = -\lambda \left(\frac{\mathrm{d}T}{\mathrm{d}z}\right)_{z=0} = \frac{\lambda U_{\infty}}{\nu (T_{0} - T_{\infty})} \left(\frac{\mathrm{d}\theta}{\mathrm{d}\zeta}\right)_{\zeta=0} . \quad (35)$$

Table 1. Values of X, Y and δ for $S_1 = 1, E_1 = 0.5$								
M = 5.00				M = 10.00				
ωτ	X	δ	Y	X	δ	Y		
0.5	5.1527	0.1941	1.1821	9.7295	0.1028	2.2211		
1.0	4.4572	0.2244	1.7057	8.3053	0.1204	3.2670		
1.5	3.8752	0.2580	1.8574	7.1129	0.1406	3.5653		
2.0	3.4528	0.2896	1.8626	6.2436	0.1602	3.5692		
2.5	3.1463	0.3178	1.8178	5.0607	0.1976	4.1656		

It may be noticed that the shear stress due to the primary flow *u* decreases and that due to the cross-flow *v* first increases, reaches a maximum and then decreases with increase in the Hall parameter $\omega \tau$ for fixed *M*. The Ekman layer thickness δ increases with increase in $\omega \tau$ for fixed *M* but decreases with increase in the strength of the magnetic field for fixed $\omega \tau$. Similar results are also found for the case of blowing at the plate.

HEAT TRANSFER

We shall now determine the temperature distribution and heat transfer in the flow for the case of suction at the plate. The equation of energy is

$$\rho C_p w_0 \frac{dT}{dz} = \lambda \frac{d^2 T}{dz^2} + \rho v \left[\left(\frac{du}{dz} \right)^2 + \left(\frac{dv}{dz} \right)^2 \right]$$
(29)

where the temperature T is taken as a function of z only and the last term in (29) represents viscous dissipation, λ being the thermal conductivity. Introducing

$$\theta(\zeta) = \frac{T - T_0}{T_\infty - T_0}, \quad Ec = \frac{U_\infty^2}{C_p(T_0 - T_\infty)}, \quad Pr = \frac{\rho \nu C_p}{\lambda} \quad (30)$$

in (29), we get on using (23) and (24)

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}\zeta^2} + Pr \cdot S_1 \frac{\mathrm{d}\theta}{\mathrm{d}\zeta} = Ec \cdot Pr \left[\left(\frac{S_1 + \alpha}{2} \right)^2 + \frac{\beta^2}{4} \right] \cdot e^{-(S_1 + \alpha)\zeta}$$
(31)

where T_0 and T_{∞} denote the temperature of the plate and the free stream respectively. We also assume $T_0 > T_{\infty}$ so that the Eckert number Ec > 0. The boundary conditions for $\theta(\zeta)$ are clear

$$\theta(0) = 0, \qquad \theta(\infty) = 1. \tag{32}$$

Both (33) and (34) give

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}\zeta}\right)_{\zeta=0} = Pr\left[S_1 - \frac{Ec\{(S_1+\alpha)^2 + \beta^2\}}{4(S_1+\alpha)}\right]. \quad (36)$$

Since $T_0 > T_{\infty}$, it follows from (35) and (36) that heat will flow from the plate to the liquid if

$$S_1 > Ec \cdot \frac{\{(S_1 + \alpha)^2 + \beta^2\}}{4(S_1 + \alpha)}$$
(37)

while heat will flow from the liquid to the plate if

$$S_1 < Ec \cdot \frac{\left[(S_1 + \alpha)^2 + \beta^2 \right]}{4(S_1 + \alpha)}.$$
(38)

It is also clear that there will be no heat transfer from or towards the wall when

$$S_1 = Ec[(S_1 + \alpha)^2 + \beta^2]/4(S_1 + \alpha).$$

Substitution of the values of α and β from (20) and (21) in (38) leads to

$$\begin{pmatrix} 1 - \frac{Ec}{4} \end{pmatrix} S_1^2 + \frac{(2 - Ec)S_1}{2\sqrt{2}} \\ \times \left[\{ (S_1^2 + 4\alpha_1)^2 + 4(4E_1 + 2\alpha_1\omega\tau)^2 \}^{\frac{1}{2}} + S_1^2 + 4\alpha_1 \right]^{\frac{1}{2}} \\ < \frac{Ec}{4} \left[(S_1^2 + 4\alpha_1)^2 + 4(4E_1 + 2\alpha_1\omega\tau)^2 \right]^{\frac{1}{2}}.$$
(39)

It is interesting to note from (38) that heat may flow from the liquid to the wall even if the wall temperature is higher than the free stream temperature T_{∞} . This will happen when increased viscous dissipation near the plate may lead to a temperature distribution in its neighbourhood greater than T_0 despite the fact $T_0 > T_{\infty}$. In fact (39) shows that heat always flows from the liquid to the plate if Ec > 4. The inequality (39) can be written as

$$Ec > \frac{A}{B} \tag{40}$$

where

$$A = S_1^2 + \frac{S_1}{\sqrt{2}} \left[\{ (S_1^2 + 4\alpha_1)^2 + 4(4E_1 + 2\alpha_1\omega\tau)^2 \}^{\frac{1}{2}} + S_1^2 + 4\alpha_1 \right]^{\frac{1}{2}}, \quad (41)$$
$$B = \frac{S_1^2}{2} + \frac{\left[(S_1^2 + 4\alpha_1)^2 + 4(4E_1 + 2\alpha_1\omega\tau)^2 \right]^{\frac{1}{2}}}{2}$$

4 4 4
$$+ \frac{S_1}{2\sqrt{2}} \left[\left\{ (S_1^2 + 4\alpha_1)^2 + 4(4E_1 + 2\alpha_1 \omega \tau)^2 \right\}^{\frac{1}{2}} + S_1^2 + 4\alpha_1 \right]^{\frac{1}{2}} . \quad (42)$$

In the absence of rotation $(E_1 = 0)$, the above inequality shows that heat will flow from the liquid to the plate if

$$Ec > \left(\frac{A}{B}\right)_{E_1=0}.$$
 (43)

A little calculation will show that $A/B < (A/B)_{E_1=0}$ and this means that as the Eckert number increases from zero, heat will flow from the liquid to the plate at a value of *Ec* smaller than the corresponding value in the absence of rotation. This is to be expected from a physical point of view since in the presence of rotation, viscous dissipation of heat is larger than that in the absence of rotation.

In Table 2, we have shown the values of the dimensionless heat transfer rate $Pr^{-1} \cdot (d\theta/d\zeta)_{\zeta=0}$ at the plate for several values of $\omega\tau$ and for $S_1 = 1, E_1 = 0.5, M = 5$.

Table 2. Values of $Pr^{-1}(d\theta/d\zeta)_{\zeta=0}$ for $S_1 = 1$. $E_1 = 0.5, M = 5$

					······································					
ωτ										
Ec	0.5	1.0	1.5	2.0	2.5					
0.3	0.1864	0.2335	0.2852	0.3314	0.3705					
0.5	-0.3560	-0.2775	-0.1914	-0.1144	-0.0491					

Thus we find that for fixed Ec, heat-transfer rate increases with increase in $\omega\tau$ and it is interesting to note that as Ec increases from 0.3 to 0.5, the direction of heat flux is reversed indicating that there exists some value of Ec for which the rate of heat transfer will be zero, as already pointed out.

DISCUSSION

It should be noted that steady asymptotic solution for the temperature distribution is possible only for the case of suction at the heated wall. In fact the conduction of heat away from the wall is balanced by the convection of heat towards the wall by suction. But for the case of blowing, no steady temperature field is possible since the liquid at infinity is progressively heated both by diffusive processes and by convection of heat towards infinity by blowing.

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ECOULEMENT ET TRANSFERT THERMIQUE DANS LA COUCHE HYDROMAGNETIQUE D'EKMAN SUR PLAQUE POREUSE AVEC EFFET HALL

Résumé—L'écoulement dans la couche d'Ekman d'un liquide conducteur sur une plaque poreuse infinie est étudié dans le cas où le liquide pénètre sous l'effet d'un champ magnétique transversal en tenant compte de l'effet Hall. On trouve qu'une solution asymptotique existe aussi bien en présence d'un soufflage que d'une aspiration sur la plaque. Pour des paramètres magnétiques et d'aspiration fixés, le frottement pariétal de l'écoulement primaire augmente avec le paramètre de Hall $\omega \tau$, tandis que celui relatif à l'écoulement secondaire augmente d'abord, atteint un maximum et ensuite décroit lorsque $\omega \tau$ augmente. Une distribution stationnaire de température dans l'écoulement, n'existe que dans les cas d'une aspiration sur la plaque et de grands nombres d'Eckert; la convection thermique s'effectue du liquide vers la paroi, même si la température de paroi est supérieure à celle de l'écoulement ambiant.

STRÖMUNG UND WÄRMEÜBERGANG IN EINER HYDRO-MAGNETISCHEN EKMANSCHICHT AN EINER PORÖSEN PLATTE MIT HALLEFFEKTEN

Zusammenfassung—Es wird die Strömung einer Ekmanschicht einer leitenden Flüssigkeit an einer unendlich langen porösen Platte untersucht, wobei die Flüssigkeit durch ein Magnetfeld quer durchdrungen wird und Halleffekte Berücksichtigung finden. Es zeigte sich, daß asymptotische Lösungen vorhanden sind sowohl für Absaugung als auch für Ausblasung aus der Platte. Bei fixierten Magnet- und Absaugungsparametern nimmt die Wandreibung der Primärströmung zu mit Zunahme der Hallparameter $\omega \tau$ während sie für die Sekundärströmung erst zunimmt, ein Maximum erreicht und dann abnimmt mit zunehmendem $\omega \tau$. Stationäre Temperaturverteilung in der Strömung existiert nur für Absaugung an der Platte und bei großen Eckwert-Zahlen. Wärme fließt von der Flüssigkeit zur Wand, selbst bei Wandtemperaturen, die höher liegen als die der umgebenden Strömung.

ТЕЧЕНИЕ И ПЕРЕНОС ТЕПЛА В ГИДРОМАГНИТНОМ СЛОЕ ЭКМАНА НА ПОРИСТОЙ ПЛАСТИНЕ С УЧЕТОМ ЭФФЕКТОВ ХОЛЛА

Аннотация — Исследовалось течение в слое Экмана при обтекании бесконечной пористой пластины электропроводящей жидкостью с наложением поперечного магнитного поля и учетом эффектов Холла. Найдено, что асимптотическое решение существует при наличии как отсоса, так и вдува на пластине. При определенных значениях параметров магнитного поля и параметров вдува поверхностное трение основного потока увеличивается с увеличением параметра Холла $\omega \tau$, в то время как вторичное течение сначала увеличивается, достигает максимума, а затем уменьшается с увеличением $\omega \tau$. Стационарное распределение температуры в потоке имеет место только для отсоса на пластине и при больших значениях числа Эккерта; поток тепла направлен от жидкости к стенке, даже если температура стенки выше температуры потока.